Baryon vertices in AdS black hole backgrounds

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We numerically computed the energy of QCD string junctions in non-supersymmetric SU(N) gauge theories as the energy of brane configurations in the background of AdS black hole solutions, and extracted the contribution of baryon vertices. We obtain a negative vertex energy for the three- and four-dimensional Yang-Mills theories and a vanishing vertex energy for the five-dimensional theory.

§1. Introduction

Although string theory was originally constructed for the purpose of describing the behavior of quarks in hadrons, it has been developed for long time independently of hadron physics as a possible form for a theory unifying gravity and other interactions, because it can be formulated only in 10 or 26 spacetime dimensions, and spin 2 massless particles arise naturally. The scale of the fundamental string tension, $1/(2\pi\alpha')$, which is near the Planck scale, was thought to have no relation to the QCD scale $1/\alpha'_{\mathrm{QCD}} \sim \mathrm{GeV}^2$.

The discovery of the AdS/CFT correspondence[1]–[3] changed this situation. In this correspondence, string theory in a $d + 1$-dimensional AdS or AdS-like spacetime, which is usually accompanied by an internal space, is dual to a gauge theory in the d-dimensional boundary of the spacetime. This enables us to study the non-perturbative dynamics of gauge theories with the help of string theory. The properties of the gauge theory are reflected by the structure of the $d+1$-dimensional spacetime and the internal space. The dual geometry of the confining gauge theory is a kind of (Wick-rotated) black hole, and the QCD strings in the boundary theory are identified with fundamental strings in the region near the horizon of the black hole. (The term "horizon" here is not used in its ordinary sense, because of the Wick rotation. On the "horizon", some space-like components of the metric vanish, while the time component g_{tt} is regular.) The difference between the QCD string tension, $1/(2\pi\alpha'_{\mathrm{QCD}})$, and the fundamental string tension, $1/(2\pi\alpha')$, is due to a large red-shift associated with the curved background.

In this paper, we consider large N SU(N) pure Yang-Mills theory in $d = 3$ and 4 dimensions. The AdS dual of this theory is the $d+1$-dimensional AdS Schwarzchild black hole solution.[4] We also consider the $d = 5$ case, for reasons discussed in §4. The 5-dimensional gauge theory is non-renormalizable, and we define it as the low-energy effective theory of open strings on D5-branes wrapped on non-supersymmetric $\mathbf{S}^1$.

Let $g[C]$ denote the path-ordered integral defined for a general open contour C

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by

$$g[C] = \mathrm{P} \exp \int_C A \in \mathrm{SU}(N). \tag{1·1}$$

If the two ends of C coincide, the Wilson loop associated with the closed loop C and the SU(N) representation R is defined by $W_R[C] \equiv \langle \rho_R(g[C])^a{}_a \rangle$, where $\rho_R(g)^a{}_b$ is the representation matrix of $g \in \mathrm{SU}(N)$ in the representation R. Let us take a rectangle of width L along the spatial direction and length t along the temporal direction as the contour C. If the charge associated with the representation R is confined, the free energy Γ defined by $W_R = e^{-t\Gamma}$ for large t takes the form

$$\Gamma = LT + 2a + \mathcal{O}(L^{-1}). \tag{1·2}$$

This can be regarded as the energy of a "meson" consisting of the charges R and $\overline{R}$ connected by a QCD string. The quantity T is interpreted as the tension of the QCD string, and a represents the boundary correction for each end of the string. The representation R is conveniently expressed in the form of by a Young tableau. Due to the vacuum polarization of the gluon field, information concerning the shape of this Young tableau is screened, and only the mod N part of the number of boxes in the tableau can be observed from a distance. This number is called the N-ality of the representation R. We refer to strings associated with a representation of an N-ality k as "k-strings."

Let $T(k)$ denote the tension of a k-string. It has a non-linear dependence on k non-linearly, and it satisfies the relations

$$T(0) = 0, \quad T(k + nN) = T(k), \quad T(k) = T(-k). \tag{1·3}$$

The last relation here follows from the fact that $(-k)$-strings are k-strings with the opposite orientation. Let us assume that the function $T(k)$ is convex except at $k = 0$ ($\mod N$). This is natural because $T(0) = T(N) = 0$ and $T(k) > 0$ for $0 < k < N$. The convexity assumption implies the inequality

$$T(k_1) + T(k_2) > T(k_1 + k_2). \quad (k_1, k_2 \neq 0 \mod N) \tag{1·4}$$

This means that two QCD strings form a truly bound state. Thus we can regard a k-string as a bound state of k 1-strings.

Through the AdS/CFT correspondence, a Wilson loop with k-ality ± 1 is mapped to a fundamental string with boundary C, which is embedded in the boundary of the AdS space. For the purpose of obtaining the QCD string tension with the help of AdS/CFT, we consider a long string most of which lies on the horizon. The QCD string tension is the fundamental string tension multiplied by the red-shift factor at the horizon. This analysis was first performed for the non-confining $\mathcal{N} = 4$ Yang-Mills theory in Ref. 5), and it was confirmed that there is no linear term. Such Wilson lines were also studied in Ref. 6) in a different way. The analysis in confining theories was first carried out in Ref 4), and a non-vanishing QCD string tension was obtained. Only the horizon in the spacetime is important when we compute the QCD string tension. We call the horizon "IR subspace."

For a large class of solutions dual to confining field theories, the IR subspace has the common structure $M_d \times \mathbf{S}^p$, where M_d is the d-dimensional flat Minkowski space and $\mathbf{S}^p$ is a p-dimensional sphere. The internal sphere $\mathbf{S}^p$, through which flux passes, plays an important role when we consider objects dual to k-strings with general k. In Ref. 7) it was shown that k coincident fundamental strings are expanded into a Dp-brane tube due to the Myers effect,[8] and the tension of k-strings is equal to the energy of the Dp-brane tube per unit length.

The meson configuration discussed above can easily be generalized to general hadron configurations. For example, we can construct a baryon configuration consisting of three charges R_i $(i = 1, 2, 3)$ in the following way. Let C_i $(i = 1, 2, 3)$ be three open contours sharing the same endpoints. Here we assume that the endpoints are at $(t, \vec{r}) = (0, \vec{0})$ and $(t, \vec{0})$, and that each C_i consists of three segments, $(0, \vec{0})$-$(0, \vec{r}_i)$, $(0, \vec{r}_i)$-$(t, \vec{r}_i)$ and $(t, \vec{r}_i)$-$(t, \vec{0})$. We define the SU(N)-valued operator $g[C_i]$ by (1·1) for each C_i, and we combine them to form the gauge singlet observable

$$B = \langle T_{abc} T^{*a'b'c'} \rho_{R_1}(g[C_1])^a{}_{a'} \rho_{R_2}(g[C_2])^b{}_{b'} \rho_{R_3}(g[C_3])^c{}_{c'} \rangle, \qquad (1·5)$$

where T_{abc} is an SU(N) invariant tensor whose three indices correspond to the representations R_1, R_2 and R_3, and $T^{*a'b'c'}$ is its complex conjugate. If all the charges R_i are confined, three QCD strings associated with these charges arise. They are attached to the contours C_i at one end, and the other ends of the three strings are joined at a so-called baryon vertex. These three strings possess the string charges k_i, which are the N-alities of the representations R_i. The existence of the invariant tensor T_{abc} requires that k_i satisfy the condition

$$k_1 + k_2 + k_3 = 0. \quad (\mod N) \qquad (1·6)$$

This can be interpreted as the mod N conservation law of the string charge. We conjecture that the free energy of this baryon configuration defined by $B = e^{-t\Gamma}$ takes the form

$$\Gamma = \sum_{i=1}^{3} (L_i T(k_i) + a(R_i)) + E_{\text{vertex}}(k_1, k_2, k_3) + \mathcal{O}(L_i^{-1}), \qquad (1·7)$$

where $T(k_i)$ is the string tension of the i-th string, and $a(R_i)$ is the boundary correction associated with the contour C_i. These two kinds of quantities are the same as those in the meson energy (1·2). The quantity L_i is the length of the i-th string. Note that this length may be different from $|\vec{r}_i|$, because the baryon vertex is in general not at the origin. The position of the vertex is kinematically determined by the balance of the tensions of the three strings.

The E_{vertex} in (1·7) denotes the contribution of the baryon vertex, and it is a new ingredient, which is absent in the meson case. On the gravity side, the object corresponding to a baryon vertex is a Dp-brane wrapped on the internal $\mathbf{S}^p$.[9],[10] Because both QCD strings and baryon vertices are realized with Dp-branes, the entire system of a junction is dual to a smooth Dp-brane configuration. Thus, it is possible to compute the energy of junctions and extract E_{vertex} from this energy

by using the Born-Infeld action and the Chern-Simons action of the Dp-brane. In a previous work of the present author,[12] E_{vertex} was computed for the Maldacena-Núñez and Klebanov-Strassler backgrounds, both of which are dual to $\mathcal{N}=1$ large N confining gauge theories. In this paper, we report the result of the same analysis for different backgrounds without supersymmetry.

§2.   AdS black hole solution

In order to realize the d-dimensional pure SU(N) Yang-Mills theory, we consider N coincident Dd-branes wrapped on a non-supersymmetric $\mathbf{S}^1$ cycle.[4] A classical solution in Type II supergravity corresponding to this brane configuration is obtained by applying double Wick rotation to the non-extremal Dd-brane solution constructed in Ref. 11), and its near horizon geometry is the AdS Schwarzschild black hole. The metric in the string frame is

$$ds^2 = f_-^{\frac{1}{2}}\eta_{ij}dX^i dX^j + f_+ f_-^{-\frac{1}{2}}dy^2 + f_+^{-1} f_-^{\frac{7-3p}{2(p-1)}}dr^2 + f_-^{\frac{5-p}{2(p-1)}}r^2 d\Omega_p^2, \qquad (2\cdot1)$$

where $p = 8 - d$, and $f_\pm \equiv f_\pm(r)$ are the following functions of r:

$$f_\pm(r) = 1 - \frac{r_\pm^{p-1}}{r^{p-1}}. \qquad (2\cdot2)$$

The two parameters r_- and r_+ represent the locations of the inner and outer horizon, respectively, and they satisfy the relation

$$\sqrt{r_+^{p-1} r_-^{p-1}} = \frac{N g_{\text{str}}^\infty (2\pi l_s)^{p-1}}{(p-1)\Omega_p}, \qquad (2\cdot3)$$

where we define l_s by $l_s \equiv \alpha'^{1/2}$, and g_{str}^∞ denotes the string coupling constant in the asymptotic region, $r \to \infty$. The coordinate y is periodic, and its period is determined by requiring the solution to be smooth at the horizon, $r = r_+$. Let the period be $2\pi R_y$. Then the Kaluza-Klein radius R_y is given by

$$R_y = \frac{2}{p-1}\frac{r_+^{(p-1)/2}}{\Delta^{(p-3)/2}}, \qquad (2\cdot4)$$

where Δ is defined by $\Delta^{p-1} \equiv r_+^{p-1} - r_-^{p-1}$. The r dependence of the dilaton field in the classical solution is given by

$$e^\phi = g_{\text{str}}^\infty f_-^{\frac{5-p}{4}}(r). \qquad (2\cdot5)$$

There exists the non-vanishing RR flux in $\mathbf{S}^p$

$$G_p = \frac{N}{\Omega_p}\omega_p, \qquad (2\cdot6)$$

where Ω_p is the volume of the unit $\mathbf{S}^p$, and ω_p is the volume form on $\mathbf{S}^p$ normalized such that $\oint \omega_p = \Omega_p$.

QCD strings in the boundary gauge theory are identified with fundamental strings or D-branes possessing the fundamental string charge in this spacetime. Because of the gravitational force, they fall to the bottom, $r = r_+$, in this spacetime. Thus, only the subspace $r = r_+$, which we call the IR subspace, is relevant to our analysis. The metric of the IR subspace is

$$ds^2 = -dt^2 + R^2((dx^a)^2 + d\Omega_p^2), \quad R = f_-^{\frac{5-p}{4(p-1)}}(r_+)r_+, \tag{2·7}$$

where we rescale the coordinates X^i in (2·1) and denote the temporal and spatial coordinates by t and x^a $(a = 1, \ldots, p)$, respectively.

In order to decouple gravity from the boundary gauge theory, we need to take the decoupling limit, $l_s \to 0$, keeping the parameters in the gauge theory fixed. If we require the QCD string tension to remain finite in this limit, the redshift factor $f_-^{1/2}(r_+)$ in the metric (2·1), which relates the fundamental string tension $\sim 1/\alpha'$ and the QCD string tension $\sim 1/\alpha'_{\rm QCD}$, should behave as

$$(g_{tt}(r_+))^2 = f_-(r_+) = 1 - \frac{r_-^{p-1}}{r_+^{p-1}} \sim \frac{\alpha'^2}{\alpha'^2_{\rm QCD}} \to 0. \tag{2·8}$$

§3.   QCD strings from AdS/CFT

The mod N nature and the formation of bound strings can be understood on the AdS side by taking account of the transition between fundamental strings and Dp-branes in the background RR flux (2·6). In the IR subspace, fundamental strings are blown up to Dp-brane tubes with electric flux on their world volumes by the Myers effect.[8] The energy of a Dp-brane is given by

$$E = T_{{\rm D}p}R^p \int d^p\sigma \sqrt{\det g_{ab}}\sqrt{1 + g_{ab}D^aD^b}, \tag{3·1}$$

where g_{ab} and D^a $(a, b = 1, \ldots, p)$ are the induced metric and the electric flux density, respectively, on the Dp-brane worldvolume. We factor out the radius R from the metric to simplify the equations in what follows. The normalization of D^a is chosen so that the expression for the D-brane energy is simplified as (3·1). The energy (3·1) does not include parameters for the background geometry, except for an overall factor that does not affect the equations of motion of the Dp-branes.

A Dp-brane tube has a cross section $\mathbf{S}^{p-1}$ wrapped on a contractible cycle in $\mathbf{S}^p$. The behavior of this cross section is analogous to that of a flexible superconducting ring in a magnetic flux background. In general, electric current exists on the ring. In this analogy, the role of the background magnetic field is played by the RR flux, and the role of the electric current on the ring by the electric flux D^a on the Dp-brane tube. If the cross section of the D-brane tube (which is topologically $\mathbf{S}^{p-1}$ in the internal space $\mathbf{S}^p$) changes, the electric flux D^a changes due to an effect analogous to the electromagnetic induction. In this process, the quantity

$$\frac{k}{N} = \frac{b}{(p-1)\Omega_p} \oint_{\mathbf{S}^{p-1}} D^a dS_a + \frac{1}{\Omega_p} \int_{\mathbf{D}^p} \omega_p \tag{3·2}$$

is invariant, where the parameter b is defined by

$$b = (p-1)\Omega_p \frac{T_{Dp}R^{p-1}}{NT_{\mathrm{str}}}. \qquad (3\cdot3)$$

The first term in (3·2) gives the total electric flux passing through the cross section of the D-brane tube and the second term is the volume enclosed by the cross section. The quantity k defined by (3·2) is identified with the string charge. Indeed, if the tube shrinks down to a string and the second term in (3·2) vanishes, k is simply the amount of the electric flux that couples to the NS-NS 2-form field. The mod N nature is reflected in the ambiguity in the definition of the inside of the cross section, due to the nontrivial homotopy $\pi_p(\mathbf{S}^p) = \mathbf{Z}$.

The equation (3·2) not only defines the charge k but also places a constraint on the electric flux. The differential form of (3·2) is $(b/(p-1))d * D + \omega_p = 0$, and this is the Gauss constraint imposed on D^a. Stable brane configurations are determined by minimizing the energy (3·1) under this constraint.

The parameter b is the only parameter of the background spacetime relevant to our analysis. Roughly speaking, this represents the "toughness" of Dp-branes in the following sense. To analyze a baryon configuration, we need to consider a baryon vertex joining strings, as mentioned in the introduction. The vertex is a Dp-brane wrapped on the $\mathbf{S}^p$.[9),10)] It is deformed by the tension of the strings attached to it. If the number of strings is of order N, the tension pulling on the Dp-brane is of the order of NT_{str}. The scale of the deformation L is determined by the ratio between this tension and the D-brane tension $T_{\mathrm{D}p}$ as $L^{-(p-1)} = T_{Dp}/(NT_{\mathrm{str}})$. If we normalize this with the size of the $\mathbf{S}^p$, we obtain the dimensionless parameter b defined above. If b is very large, the Dp-brane is "tough", and the deformation is negligible compared to the size of the baryon vertices, while if b is of order 1 or larger, the brane is "soft", and the deformation caused by the string tension should be taken into account.

Let us determine the value of the parameter b for the IR subspace of the AdS black hole solution in §2. The fundamental string tension and the Dp-brane tension in the IR subspace are

$$T_{\mathrm{str}} = \frac{1}{2\pi l_s^2}, \quad T_{\mathrm{D}p} = \frac{1}{(2\pi)^p l_s^{p+1} g_{\mathrm{str}}^{\mathrm{hor}}}, \qquad (3\cdot4)$$

where $g_{\mathrm{str}}^{\mathrm{hor}} = g_{\mathrm{str}}^\infty f_-^{\frac{5-p}{4}}(r_+)$ is the string coupling constant at $r = r_+$. Substituting these and R in (2·7) into the definition (3·3) of the parameter b, we obtain

$$b = \left(\frac{r_+}{r_-}\right)^{\frac{p-1}{2}}. \qquad (3\cdot5)$$

In the decoupling limit (2·8), this becomes 1 for any value of p. Thus we have to take account of the deformation of D-branes.

Now we are ready to compute the QCD string tensions for $p = 3, 4, 5$. Let θ be the angular radius of the cross section of the tube. For given k, the energy of the

Dp-brane tube is minimized at the value of θ satisfying

$$\frac{k}{N} = \frac{\Omega_{p-1}}{\Omega_p} \left[\frac{b^2}{p-1} \cos\theta \sin^{p-2}\theta + \int_0^\theta \sin^{p-1} x\, dx \right], \qquad (3\cdot6)$$

and the tension is given by

$$T(k) = T_{\mathrm{D}p}\Omega_{p-1}R^{p-1}\sin^{p-2}\theta\sqrt{\sin^2\theta + b^2\cos^2\theta} = T_{\mathrm{D}p}\Omega_{p-1}R^{p-1}\sin^{p-2}\theta, \quad (3\cdot7)$$

where we have used $b = 1$ in the final step. The tension for $p = 3,\ 4,\ 5$, is given as follows:

$$T_{p=3}(k) = T_{\mathrm{D}3}\Omega_2 R^2 \sin\left(\frac{\pi k}{N}\right), \qquad (3\cdot8)$$

$$T_{p=4}(k) = T_{\mathrm{D}4}\Omega_3 R^3 \frac{4k(N-k)}{N^2}, \qquad (3\cdot9)$$

$$T_{p=5}(k) = T_{\mathrm{D}5}\Omega_4 R^4 \sin^3\theta$$
$$\text{with } \theta \text{ solving} \quad \pi(k/N) = \theta - \cos\theta\sin\theta. \qquad (3\cdot10)$$

(In order to obtain finite QCD string tensions in the decoupling limit, we need to rescale these tensions by appropriate warp factors. We here do not include them, because we are interested in the relative magnitude of the vertex energies and the energies of the corresponding wrapped branes.) A plot of the function $T(k)$ is given in Fig. 1. As seen there, the functions $T(k)$ for different p are quite similar.
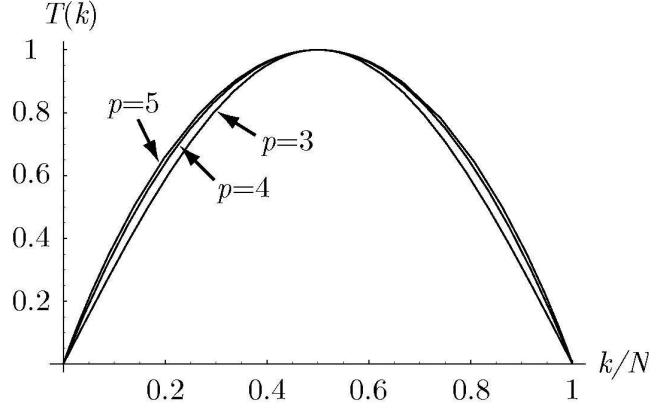

Fig. 1. The k dependence of the string tension for $p = 3, 4, 5$. We normalize the tension such that $T(N/2) = 1$.

§4. Numerical analysis of junctions

In this section, we report the results of numerical computation of the energy of baryon vertices for $p = 3, 4, 5$ carried out with the following steps.
- Firstly, we determine the directions ϕ_i ($i = 1, 2, 3$) of the three strings in a junction by imposing the condition that the three string tensions balance. If

the charges of the three strings are k_i, the angles ϕ_i are determined by

$$\cos(\phi_2 - \phi_3) = \frac{T^2(k_2) + T^2(k_3) - T^2(k_1)}{2T(k_2)T(k_3)}, \qquad (4\cdot1)$$

and similar equations obtained by permuting the subscripts 1, 2 and 3.

- Secondly, we construct an initial configuration consisting of three strings for our numerical computation. The three strings are joined at one end of each string by a baryon vertex, and the other ends are pinned on a circle with a sufficiently large radius L. We use $L = 4R$ in the analysis below. The positions on the circle are chosen as $(L\cos\phi_i, L\sin\phi_i)$ $(i = 1, 2, 3)$, using the angles obtained in the first step, so that the baryon vertex coincides with the center of the circle.
- Finally, we determine the brane configurations minimizing the energy. Then we extract the energy E_{vertex} of vertex from the energy E_{junc} of the junction using the relation

$$E_{\text{vertex}} = E_{\text{junc}} - \sum_{i=1}^{3} LT(k_i). \qquad (4\cdot2)$$

As preparation for the numerical analysis, we should reduce the problem to a lower dimensional one. We consider only planar junctions in the (x, y)-plane. We embed the internal space, $\mathbf{S}^p$, into $\mathbf{R}^{p+1}$ using the Cartesian coordinates $(u, v, w_1, \ldots, w_{p-1})$. Then, we assume that the Dp-brane worldvolumes, which are branched tubes with cross section $\mathbf{S}^{p-1}$, are invariant under the $SO(p-1)$ rotation in the $(w_1, \ldots, w_{p-1})$-space, and we represent a worldvolume as an $SO(p-1)$ orbit of a two-dimensional surface in the (x, y, u, v)-space. Next, we introduce the flux density D'^a on the surface and the surface integral measure dn_a by

$$D'^a = w^{p-2}D^a \quad (a = u, v), \quad dS_a = \Omega_{p-2}w^{p-2}dn_a, \qquad (4\cdot3)$$

where $w = \sqrt{\sum_i w_i^2} = \sqrt{1 - u^2 - v^2}$. The energy of a brane and the integral form of the Gauss law constraint for the electric flux are given by

$$\frac{E}{T_{\text{D}p}R^p} = \Omega_{p-2} \int d^2\sigma \sqrt{\det g_{ab}} \sqrt{w^{2(p-2)} + g_{ab}D'^a D'^b}, \qquad (4\cdot4)$$

$$2\pi\frac{k}{N} = b \int dn_a D'^a + (p-1) \int w^{p-3}du \wedge dv. \qquad (4\cdot5)$$

The problem we consider is to find two-dimensional surfaces that minimize the energy (4·4) under the constraint (4·5), and then to compute these minimal energies. We realize the two-dimensional surface as a mesh consisting of triangles. We give only the result below. (See Ref.12) for the details of the numerical analysis.)

4.1. $p = 3$

The classical solution with $p = 3$ describes the near-horizon geometry of coincident D5-branes wrapped around non-supersymmetric $\mathbf{S}^1$. Because the five-dimensional gauge theory is non-renormalizable, it should be regarded as the low-energy effective theory of open strings on the D5-branes.

There are other reasons that the $p = 3$ case is interesting. By performing the S-duality transformation, we obtain the NS5-brane solution, and its near horizon geometry is

$$ds^2 = \eta_{ij}dX^i dX^j + r_+^2 \left[h(\rho)d\theta^2 + h^{-1}(\rho)\frac{d\rho^2}{\rho^2} + d\Omega_3^2 \right], \quad g_{\text{str}} = g_{\text{str}}^\infty \frac{r_+}{\rho}, \qquad (4\cdot6)$$

where $h(\rho) = 1 - \Delta^2/\rho^2$, and we have introduced the angular variable $\theta = y/r_+$ with period 2π. It is known that string theory in this background is solvable. The two-dimensional space parameterized by (θ, ρ) constitutes a cigar geometry and is described by the supersymmetric level N $\text{SL}(2, \mathbf{R})/\text{U}(1)$ CFT with central charge $c = 3(N + 2)/N$. The $\mathbf{S}^3$ part can be described by the supersymmetric $\text{SU}(2)_N$ WZW model with the central charge $c = 3(N - 2)/N + 3/2$.

Another reason to study the $p = 3$ case is that the vertex energy in this case can be computed analytically. The IR subspace with $p = 3$ and $b = 1$ is also regarded as the IR subspace of the Maldacena-Núñez solution,[13),14)] which is the near horizon geometry of the D5-branes wrapped around the 2-cycle of a conifold. The Maldacena-Núñez solution is believed to be dual to the $\mathcal{N} = 1$ Yang-Mills theory. In Ref. 12), it was analytically shown that the energies of vertices in this case vanish. Although we know the exact result, we present the result of the numerical analysis in order to clarify the accuracy of the method. We use $R^3 T_{\text{D3}}$ as the unit of energy. We find

Table I. The energies of baryon vertices for $p = 3$ and $b = 1$. The first column lists the ratio of charges of the three strings in a junction. The next three columns list the energies of the junctions realized as triangular meshes consisting of $n_{\text{mesh}} = 6 \times 20^2$, $n_{\text{mesh}} = 6 \times 40^2$ and $n_{\text{mesh}} = 6 \times 80^2$ triangles, respectively. The energy E_∞ is obtained by extrapolating these results to the continuum limit $n_{\text{mesh}} \to \infty$. The quantity E_0 represents the energy of strings defined by $E_0 = L \sum_{i=1}^3 T(k_i)$. The last column shows the energy of vertex extracted by $E_{\text{vertex}} = E_\infty - E_0$.

$(k_1 : k_2 : k_3)$	$E_{6 \times 20^2}$	$E_{6 \times 40^2}$	$E_{6 \times 80^2}$	E_∞	E_0	E_{vertex}
$(4 : 4 : 4)$	130.3545	130.5356	130.5794	130.5939	130.5936	$+0.0003$
$(3 : 4 : 5)$	127.3866	127.5686	127.6127	127.6273	127.6270	$+0.0003$
$(2 : 5 : 5)$	121.9670	122.1716	122.2217	122.2383	122.2382	$+0.0001$

the error is of the order of $10^{-5} E_w$, where $E_w = \Omega_3 R^3 T_{\text{D3}} = 19.74$ is the energy of a wrapped brane, which represents the order of the result that we expect on the basis of naive considerations.

4.2. $p = 4$

The dual geometry of the four-dimensional pure Yang-Mills theory is obtained by setting $p = 4$ and $b = 1$, and the dual to a QCD junction is a branched D4-brane tube. The results of the numerical analysis for the energies of baryon vertices is presented in Table II. We use $R^4 T_{\text{D4}}$ as the unit of energy. It is seen that the result is always negative, and the absolute value is much smaller than the energy of a D4-brane wrapped on $\mathbf{S}^4$, $E_w = \Omega_4 R^4 T_{\text{D4}} = 26.32$. Even at the maximum point with $k_1 = k_2 = k_3$, it is only 3.321% of E_w. The charge dependence of the energy is graphically depicted in Fig. 2 (a).

Table II.   Results of numerical analysis for the baryon vertex energy in the $p = 4$ case. (Refer to the caption of Table I for the meaning of each column.)

$(k_1 : k_2 : k_3)$	$E_{6 \times 20^2}$	$E_{6 \times 40^2}$	$E_{6 \times 80^2}$	E_∞	E_0	E_{vertex}
$(4 : 4 : 4)$	208.634	209.417	209.611	209.676	210.552	-0.874
$(3 : 4 : 5)$	204.338	205.104	205.293	205.356	206.165	-0.809
$(3 : 3 : 6)$	195.734	196.441	196.616	196.674	197.392	-0.718
$(2 : 5 : 5)$	195.738	196.506	196.695	196.758	197.392	-0.634
$(2 : 4 : 6)$	191.432	192.157	192.337	192.397	193.006	-0.609
$(2 : 3 : 7)$	178.501	179.111	179.261	179.311	179.846	-0.535
$(2 : 2 : 8)$	156.907	157.363	157.474	157.511	157.914	-0.403
$(1 : 5 : 6)$	178.484	179.278	179.473	179.538	179.846	-0.308
$(1 : 4 : 7)$	169.861	170.553	170.723	170.779	171.073	-0.294
$(1 : 3 : 8)$	152.576	153.098	153.224	153.266	153.527	-0.261
$(1 : 2 : 9)$	126.591	126.904	126.977	127.002	127.208	-0.206
$(1 : 1 : 10)$	91.830	91.961	91.989	91.998	92.116	-0.118

4.3. $p = 5$

The dual geometry of the three-dimensional pure Yang-Mills theory is obtained by setting $p = 5$ and $b = 1$, and the dual to a QCD junction is a branched D5-brane tube. The results of the numerical analysis for the energies of baryon vertices is presented in Table III. We use $R^5 T_{\text{D5}}$ as the unit of energy. Here, it is found that the result is always negative and the absolute value is much smaller than the energy of a D5-brane wrapped on $\mathbf{S}^5$, $E_w = \Omega_5 R^5 T_{\text{D5}} = 31.01$. Even at the maximum point with $k_1 = k_2 = k_3$, it is only 3.325% of E_w. The charge dependence of the vertex

Table III.   Results of numerical analysis for the baryon vertex energy in the $p = 5$ case. (Refer to the caption of Table I for the meaning of each column.)

$(k_1 : k_2 : k_3)$	$E_{6 \times 20^2}$	$E_{6 \times 40^2}$	$E_{6 \times 80^2}$	E_∞	E_0	E_{vertex}
$(4 : 4 : 4)$	280.022	281.610	282.004	282.135	283.166	-1.031
$(3 : 4 : 5)$	274.516	276.067	276.451	276.579	277.542	-0.963
$(3 : 3 : 6)$	263.653	265.101	265.460	265.579	266.450	-0.871
$(2 : 5 : 5)$	263.176	264.699	265.076	265.201	265.980	-0.779
$(2 : 4 : 6)$	257.817	259.278	259.638	259.758	260.512	-0.754
$(2 : 3 : 7)$	241.629	242.906	243.220	243.324	244.001	-0.677
$(2 : 2 : 8)$	214.206	215.223	215.468	215.549	216.084	-0.535
$(1 : 5 : 6)$	239.959	241.484	241.858	241.198	242.396	-0.406
$(1 : 4 : 7)$	229.266	230.662	230.999	231.110	231.509	-0.399
$(1 : 3 : 8)$	207.701	208.811	209.078	209.166	209.530	-0.373
$(1 : 2 : 9)$	174.719	175.455	175.619	175.672	175.989	-0.317
$(1 : 1 : 10)$	129.281	129.657	129.731	129.754	129.956	-0.202

energy is graphically depicted in Fig. 2 (b).

§5.   Conclusions and Discussions

In this paper we computed the energies of baryon vertices in non-supersymmetric large N SU(N) gauge theories in $d = 3$, 4 and 5 dimensions. We realized the QCD

(a) $p=4$ (b) $p=5$

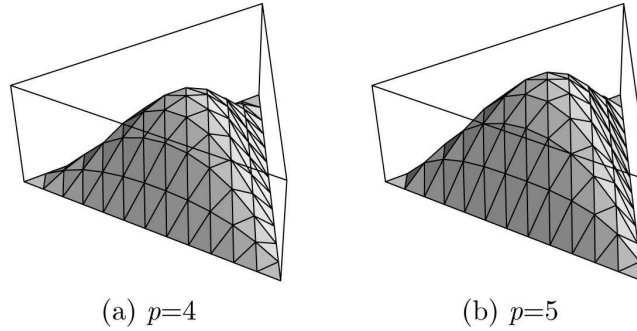Fig. 2. 3D-plot of the function $-E_{\text{vertex}}(k_1, k_2, k_3)$ for $p = 4$ and 5. Various combinations of the three string charges satisfying $k_1 + k_2 + k_3 = N$ are represented as points in the triangle. The distance from a point to any side is proportional to the corresponding string charge. The absolute value of E_{vertex} is represented by the height of the corresponding point on the surface.

junctions as D-brane configurations in AdS black hole backgrounds. In the $d = 5$ case the problem can be analytically solved, and it is found that the vertex energy vanishes. For $d = 3$ and 4, we obtained negative energies from numerical analysis. We also found that the absolute value of the energy is at most only a few percent of that of a wrapped D-brane. Even though we have obtained results on the AdS side, their physical meaning is still not clear. It would be interesting to study this problem using a more direct approach in large N gauge theory.

Because there is a large class of classical solutions appearing in AdS/CFT as the duals of confining gauge theories for which the IR subspace possesses the $M_d \times \mathbf{S}^n$ structure, we can compute the vertex energy of other theories by simply replacing the parameters p and b. In this paper, we studied only the case with $b = 1$. There are also classical solutions whose IR subspaces have $b \neq 1$. For example, the IR subspace of the Klebanov-Strassler solution[15)–17)] is obtained by setting $b = 0.93266 \cdots$ and $p = 3$. In Ref. 12), it was shown that the vertex energy in the Klebanov-Strassler solution is negative. Because we know that the vertex energy for $b = 1$ and $p = 3$ vanishes, it seems that as the parameter b increases, the vertex energy also increases. This is consistent with the fact that in the $b \to \infty$ limit (the tough D-brane limit) the vertex energy is simply the energy of a wrapped D-brane and is positive. Another example of an IR subspace with $b \neq 1$ is the IR subspace of the G_2 manifold constructed in Ref. 18). It is obtained by setting $b = 0.9249 \cdots$ and $p = 4$. The confining strings in this IR subspace were studied in Ref. 19). (The b-parameter used here is equivalent to $3A/Q$ in Ref. 19).) For the reason mentioned above, again in this case we expect a negative vertex energy.

In all the examples considered above, the vertex energy is non-negative. It may be interesting to seek classical solutions whose IR subspaces have large b, because in the gauge theories dual to such classical solutions, the QCD strings possess an interesting property. If the parameter b is larger than the critical value $b_{\text{cr}} = \sqrt{p-1}$, the extremal condition (3·6) for QCD strings has more than one solution for a certain range of k. In this case, there can be two different QCD strings of the same charge, and if $k = N/2$, the tensions of these two QCD strings are equal, and we can consider

a string consisting of two or more domains.

The analysis performed in this paper is only valid in the large N limit. When we use the results we obtained here for analysis of the real hadron spectrum, we should not regard the values in Table II as accurate predictions, because they may include errors of order $1/N = 1/3$. A lesson learned here is that we should not add the energy of a wrapped brane to the energy of a hadron, and it would be better to simply omit the vertex contribution.

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